Partial Differential Equations and Mathematical Physics
Mini Conference on
Topics in Eulers Equation for Incompressible Fluids
Wednesday May 14 Friday May 16, 2014
Talks in 127 Hayes-Healy Hall

ABSTRACTS

Continuous Dependence on the Density for Stratified Steady Water Waves
Ming Chen, University of Pittsburgh

We consider small-amplitude periodic traveling waves in a stratified water. In modeling such waves, two distinct regimes are commonly used: continuous stratification, and layer-wise continuous stratification. It can be inferred from the conservation of mass that the stratification can be defined by prescribing the value of the density on each streamline (called the streamline density function). We show that, for every smoothly stratified traveling wave in a certain small-amplitude regime, there is an L^∞ neighborhood of its streamline density function, such that, for any piecewise smooth streamline density function in that density, there is a corresponding traveling wave solution. Moreover, this mapping from streamline density function to waves is Lipschitz continuous in a certain function space framework. This is a joint work with Samuel Walsh.

Large-Data Global Well-Posedness for the (1+2)-Dimensional Equivariant Faddeev Model
Matthew Creek, University of Rochester

The Faddeev model of classical field theory is a model which describes the interactions between subatomic particles called pions. It is a generalization of the well-known classical nonlinear sigma model of Gell-Mann and Levy, and is also related closely to the celebrated Skyrme model. The Faddeev model is notable for giving rise to knotted topological solitons. The global well-posedness of the quasilinear PDE arising from this model has been studied intensely in recent years, both in three and two dimensions. In this presentation, we introduce a proof of large-data global well-posedness of the two-dimensional Faddeev model under the equivariant hypothesis.

The Free Boundary Euler Equations with Large Surface Tension
Marcelo Disconzi, Vanderbilt University

We study the free boundary Euler equations in three spatial dimensions with surface tension. Under natural assumptions, we prove that solutions of the free boundary fluid motion converge to solutions of the Euler equations in a fixed domain when the coefficient of surface tension κ tends to infinity. The well-posedness of the free boundary equations under the relevant hypothesis for the study of the limit $\kappa \to \infty$ is also established. This is joint work with David G. Ebin.

Continuity Properties of the Data-To-Solution Map for the Generalized CamassaHolm Equation
John Holmes, University of Notre Dame

We study a generalized Camassa-Holm equation with higher order nonlinearities ($g - kb\, CH$). The Camassa-Holm, the Degasperis-Procesi and the Novikov equations are integrable members of this family of equations. $g-kb\, CH$ is well-posed in Sobolev spaces H^s, $s > 3/2$, on both the line and the circle and its solution map is continuous but not uniformly continuous. We show that the solution map is Hölder continuous in H^s equipped with the H^r-topology for $0 \leq r < s$, and the Hölder exponent is expressed in terms of s and r.

On the Initial-Boundary Value Problem for the Boussinesq Equation

Dionyssis Mantzavinos, University of Notre Dame

The Boussinesq equation was introduced by J. Boussinesq in 1872 in his attempt to explain solitons. While the initial value (Cauchy) problem for this equation has been studied extensively by several researchers, this is not the case concerning initial-boundary value problems. In this talk, we will discuss well-posedness for the Boussinesq equation on the half-line by suitably adapting the approach of contraction used in the full line case.

Long-time Behaviour of Stochastic Reaction-Diffusion Equations.

Alex Mesiats, Purdue University

We study the long-time behavior of systems governed by nonlinear reaction-diffusion type equations $du = (Au + f(u))dt + \sigma(u)dW(t)$, where A is an elliptic operator, f and σ are nonlinear maps and W is an infinite dimensional nuclear Wiener process. This equation is known to have a uniformly bounded (in time) solution for $A = \Delta$ provided $f(u)$ possesses certain dissipative properties. The existence of a bounded solution implies, in turn, the existence of an invariant measure for this equation, which is an important step in establishing the ergodic behavior of the underlying physical system. In my presentation I will talk about expanding the existing class of nonlinearities f and σ, for which the invariant measure exists. We also show that the equation has a unique invariant measure if A is a Shrodinger-type operator $A = 1/\rho(div\rho\nabla u)$ where $\rho = e^{-|x|^2}$ is the Gaussian weight. In this case the source of dissipation comes from the operator A instead of the nonlinearity f. The main idea is to show that the reaction-diffusion equation has a unique bounded solution, defined for all $t \in \mathbb{R}$, i.e. that can be extended backwards in time. This solution is an analog of the trivial solution for the linear heat equation.

Persistence Properties and Unique Continuation for a Generalized Camassa-Holm Equation

Ryan C. Thompson (joint work with Alex Himonas), University of Notre Dame

Persistence properties of solutions are investigated for a generalized Camassa-Holm equation (g-kbCH) having $(k + 1)$-degree nonlinearities and containing as its integrable members the Camassa-Holm, the Degasperi-Procesi and the Novikov equations. These properties will imply that strong solutions of the g-kbCH equation will decay at infinity in the spatial variable provided that the initial data does. Furthermore, it is shown that the equation exhibits unique continuation for appropriate values of the parameters b and k. Finally, existence of global solutions is established when $b = k + 1$.

Remarks on the Regularity Criteria of Three-Dimensional MHD Systems in Terms of Two Velocity Field Components

Kazuo Yamazaki, Oklahoma State University

We discuss recent developments on the regularity criteria of the three-dimensional magnetohydrodynamics (MHD) system, which is the Navier-Stokes equations when the magnetic field vanishes. In particular, we discuss component reduction results of its regularity criteria, such as the criteria only in terms of two velocity field components dropping the condition on the third component completely. We also extend our discussion to related systems such as the magneto-micropolar fluid system and also to higher dimensions, four and five.